Plausibility Revision in Higher-Order Logic

with an Application in Two-Dimensional Semantics

Erich Rast
erich@snafu.de

IFL, Universidade Nova de Lisboa

LogKCA-10, November 2010

Instituto de Filosofia da Linguagem
Universidade Nova de Lisboa

Overview

Motivation: Why HOL?

- Why not? - Fun, curiosity, etc.
- Descriptive adquacy: Work with realistic representations of semantic content and (halfway) realistic examples.
- Investigate how (non-Gricean) notions of interpretation can be expressed in a two-dimensional setting.
- Integrate methods from formal epistemology with tools used commonly in general semantics (Montague Grammar, TLG, CCG).
- Long term goal: Express (aspects of) the interpretation of utterances on top of a traditional, two-dimensional semantic representation.

What Needs to be Done

- Implement two-dimensional semantics.
- Implement plausibility as a preorder to obtain a form of graded belief.
- Implement preorder revision.
- Apply in two-dimensional setting.
- Distinguish interpretative notions from linguistic notions.
- Implement a simple notion of interpretation:
  - Nonindexical and indexical expressions are interpreted.
  - The hearer evaluates the semantic content of a sentence on the basis of what he assumes that the speaker believes.

Simple Type Theory/HOL – Syntax

Types

Primitive types: *e* for entities, *c* for states, *t* for $\{1,0\}$. If α, β are primitive types, then $(\alpha\beta)$ is a compound type. (Parentheses are left out – right-associativity is assumed.)

Syntax

If A is a term of type $\beta\alpha$ and B is a term of type β, then (AB) is a term of type α. If A is a term of type α and x is a variable of type β then $(\lambda x A)$ is a term of type $\beta\alpha$. The identity symbol $=$ is of type $\alpha\alpha t$ for any type α.

- Standard conventions: dot notation, infix notation for common connectives, $\forall x A$ for $\forall(\lambda x A)$, leave out parentheses, e.g. $A\,x\,y$ instead of $((A\,x)\,y)$, etc.

Simple Type Theory/HOL – Semantics

Generalized Model

A collection of non-empty sets D_α for primitive types α, $D_{(\alpha\beta)} \subseteq D_\beta^{D_\alpha}$ for compound types $(\alpha\beta)$, and a term evaluation function $[\![.]\!]^g$ under assignment g.

Truth in a Model

- $[\![A^\gamma]\!]^g = g(A)$ for variable A, where $g(A) \in D_\gamma$
- $[\![A^\gamma]\!]^g \in D_\gamma$ for constant A
- $[\![(A^{\beta\alpha} B^\beta)]\!]^g = [\![A]\!]^g ([\![B]\!]^g)$
- $(\lambda x^\beta A^\alpha)$ is the function $f \in D_\alpha^{D_\beta}$ such that $[\![A^\alpha]\!]^{g[x/a]} = f(a)$ for any $a \in D_\beta$
- $[\![((=^{\alpha\alpha t} A^\alpha) B^\alpha)]\!]^g = 1$ if $[\![A^\alpha]\!]^g = [\![B^\alpha]\!]^g$ (0 otherwise)
- $[\![(\wedge A^t) B^t]\!]^g = 1$ if $[\![A]\!]^g = 1$ and $[\![B]\!]^g = 1$ (0 otherwise)
- $[\![\neg^{tt} A^t]\!]^g = 1$ if $[\![A]\!]^g = 0$ (0 otherwise)

Applicative Categorial Grammar

$$(\sigma/\tau) : A^{(\beta\alpha)}\, \tau : B^{\beta} \stackrel{f}{\Rightarrow} \sigma : (A\, B) \qquad \text{forward concatenation} \qquad (1)$$

$$\tau : B^{\beta}\, (\tau\backslash\sigma) : A^{(\beta\alpha)} \stackrel{b}{\Rightarrow} \sigma : (A\, B) \qquad \text{backward concatenation} \qquad (2)$$

Notes:

- Unlike in some other uses of CG, semantic representations are not evaluated.
- Shortcuts are possible, e.g. by different evaluation strategies (von Stechow/Zimmerman 2005), parameterizing truth in a model to contexts and indices, lifting modalities to the meta level, etc.
- For more realistic examples the full power of the Lambek calculus (hypothetical reasoning) plus other extensions like in TLG, CCG, Mortgat (1995) are needed.

Two-Dimensional Semantics

- Based on Kaplan (1989): In addition to indices for normal modal logical operators, add context parameters.
- Kaplan (1989) does not endorse structural symmetry of contexts and indices at all – but see e.g. others such as von Stechow/Zimmerman (2005).
- In the current setting: Make meanings of type $cc\alpha$, i.e. functions from an utterance 'situation' (state) to a function from a topic 'situation' (state) to an extension of type α.
- Ideally, situations would be used.
  - See Muskens (1995) for a partial, relation type theory.
  - It is prima facie not clear how to implement revision in a partial setting (future research needed).

Two-Dimensionalism: Example

Let the lexicon entries for ‘I’, ‘me’ be $np : \lambda us.speaker\, u$, for ‘Mary’ be $np : \lambda us.m$, and for ‘loves’ be $(s\backslash np)/np : \lambda ijus.love\, s\, (jus)\, (ius)$, where $speaker$ is of type ce and $love$ is of type $ceet$, where i, j are intensional types cce. Then:

$$① \text{ ‘Mary loves me’} \tag{3}$$

$$= np : \lambda us.m \; (np\backslash s)/np : \lambda ijus.love\, s\, (jus)\, (ius) \tag{4}$$

$$np : \lambda us.speaker\, u$$

$$\Rightarrow np : \lambda us.m \; np\backslash s : \lambda jus.love\, s\, (jus)\, (speaker\, u) \tag{5}$$

$$\Rightarrow s : \lambda us.love\, s\, m\, (speaker\, u) \tag{6}$$

Conditions for Binary Relations

Of course, everything can be formulated in the object language and familiar conditions on binary relations of type *cct* can be expressed as properties of relations:

$$\mathrm{Trans} := \lambda P.\forall stu[(Pst \wedge Ptu) \rightarrow Psu] \tag{7}$$

$$\mathrm{Eucl} := \lambda P.\forall stu[(Pst \wedge Psu) \rightarrow Ptu] \tag{8}$$

$$\mathrm{Ser} := \lambda P.\forall s\exists t[Pst] \tag{9}$$

$$\mathrm{Refl} := \lambda P.\forall s[Pss] \tag{10}$$

KD45 Accessibility R^{ecct}:

$$\forall x[\mathrm{Trans}(Rx) \wedge \mathrm{Eucl}(Rx) \wedge \mathrm{Ser}(Rx)] \tag{11}$$

Plausibility

Plausibility $\geq$ of type *eccct* is implemented as a preorder in the last two argument places:

$$\mathrm{Trans}(\geq xs) \tag{12}$$

$$\mathrm{Refl}(\geq xs) \tag{13}$$

with additional condition

$$\forall xup[\exists v(pv) \rightarrow \exists s(ps \wedge \neg\exists t[pt \wedge t >_{x,u} s])] \tag{14}$$

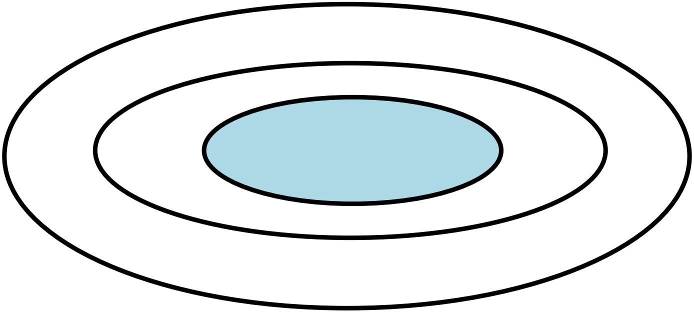
As usual,

- $s \sim_{x,u} t$ iff. $s \geq_{x,u} t$ and $t \geq_{x,u} s$,
- and $s >_{x,u} t$ iff. $s >_{x,u} t$ and not $t >_{x,u} s$.

Maximum

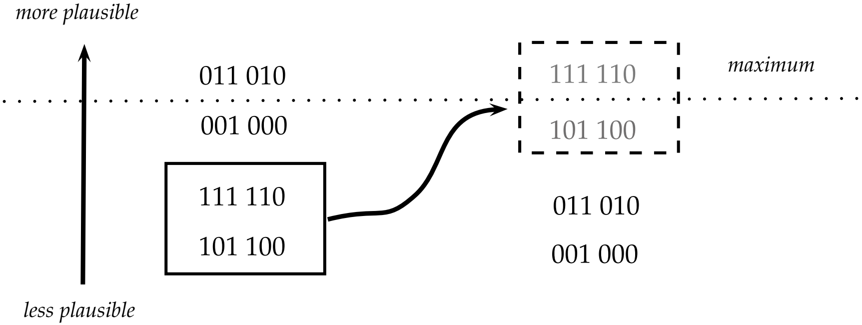
The maximum of a non-empty proposition p of type ct w. r. t. relation C of type $eccct$ is computed by:

$$\mathrm{Max} := \lambda xuCp.\iota q \forall s[(ps \wedge \neg\exists t[pt \wedge Cxuts \wedge \neg Cxust]) \equiv qs] \quad (15)$$



Plausibility Revision

Based on van der van Benthem/Liu (2005), Liu (2008), Lang/van der Torre (2008): To revise by *p*, shift all *p*-states on top of the non-*p*-states.



Plausibility Revision II

As an auxiliary notion, let '$\text{when } A^t \text{ then } B_1^t \text{ otherwise } B_2^t$ abbreviate $(A \rightarrow B_1) \wedge (\neg A \rightarrow B_2)$. The revision C' of an ordering relation C conditional on P for some agent x at u_0 is then characterized the following term.

$$\begin{aligned} \mathrm{REV} := \lambda x u_0 P C . \iota C' \forall yust [&\text{when } u_0 = u \wedge x = y \wedge Pus \wedge \neg Put \\ &\wedge Rxus \wedge Rxut \text{ then } (C'xust \wedge \neg C'xuts) \text{ otherwise } (C'yust \equiv Cyust) \\ &\wedge (C'yuts \equiv Cyuts)] \end{aligned} \tag{16}$$

Read: For given base situation u, if Pus and $\neg Put$ for two states s, t, then ensure that $s >' t$ for the revised preorder $>'$, otherwise leave the preorder unchanged.

Belief vs. Interpretative Belief

Linguistic Belief

Indexicals are not evaluated:

$$\mathrm{Bel} := \lambda x u_0 s_0 C P.\forall s_1[(\mathrm{Max}\, x s_0\, C(R x s_0))s_1 \rightarrow P u_0 s_1] \tag{17}$$

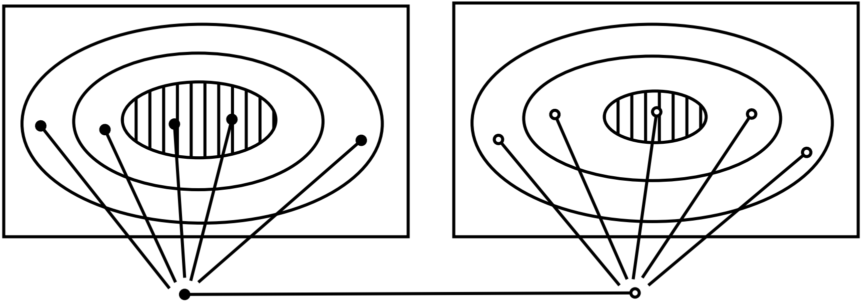
Interpretative Belief

Indexicals are evaluated:

$$\mathrm{IBL} := \lambda x u_0 s_0 C P.\forall u_1 s_1[([\mathrm{Max}\, x u_0\, C(R x u_0)]u_1 \wedge [\mathrm{Max}\, x s_0\, C(R x s_0)]s_1) \rightarrow P u_1 s_1] \tag{18}$$

Interpretative notions are analoguous to using diagonalization in a double-index modal logic, using an operator such as $M, c, i \models \Delta\phi$ iff. $M, i, i \models \phi$ when contexts and indices are structurally alike.

The Structure of an Interpretative Belief



Non-iterated Interpretative Assumptions

Weak interpretative assumptions:

$$\mathrm{IAW} := \lambda xyu_0 s_0 CP.\forall u_1 s_1 s_2[([\mathrm{Max}\, xu_0\, C(Rxu_0)]u_1 \wedge [\mathrm{Max}\, xs_0\, C(Rxs_0)]s_1 \wedge [\mathrm{Max}\, ys_1\, C(Rys_1)]s_2) \rightarrow Pu_1 s_2] \quad (19)$$

Reflect

- ...what the hearer believes about the utterance situation
- ...what the hearer believes that the speaker believes about the topic situation

Strong interpretative assumptions:

Additionally reflect what the hearer believes that the speaker believes about the utterance situation.

Using Revision

$$\begin{aligned} \mathrm{RAW} := \lambda x y u_0 s_0 P.\iota Q \forall u_1 u_2 s_1 s_2 [([\mathrm{Max}\, x u_0 \geq (R x u_0)] u_1 \\ \wedge [\mathrm{Max}\, x s_0 \geq (R x s_0)] s_1 \wedge [\mathrm{Max}\, x s_1 (\mathrm{REV}\; y s_1 P \geq)(R y s_1)] s_2) \\ \equiv Q u_1 s_2] \end{aligned} \tag{20}$$

- That meaning/2d-intension Q such that Q holds according to what x believes that y believes given that y accepts P (for given base states u_0, s_0).
- This notion is based on the revision by P of what y believes according to x's beliefs, where the utterance situation is only taken into account according to x's beliefs.
- Corresponding strong notion: Additionally it is taken into account what y believes about the utterance situation according to what x believes.

Some Form of Interpretation

$$\mathrm{IPW} := \lambda xyusPQ.\mathrm{IBL}\, xus(\mathrm{REV}\, xu(\mathrm{RAW}\, xyusP) \geq) Q \qquad (21)$$

- Speaker utters a sentence whose meaning is P (disregarding ambiguity for simplicity).
- Assumption: Speaker is not deceptive, sincere, etc.
- $\mathrm{RAW}\, xyusP$ represents what the speaker y believes given that P (according to x's beliefs).
- $\mathrm{IPW}\, a\, b\, u_0\, s_0\, P\, Q$: Q holds according to a's interpretative belief generated by his believes revised by what he believes that the speaker b believes *given that* P (in given base states u_0, s_0).
- This form of interpretation captures the hearer's interpretation of the *literal meaning* of an utterance on the basis of a model of what the speaker believes. ($\neq$ Gricean speaker meaning)

Example (informal)

John : I am here. (22)

Mary : No, you're not! You went to the Continental! (23)

- John and Mary want to meet in the lobby of the Holiday In.
- Suppose John is at the Continental and believes he's at the Holiday In.
- Suppose Mary believes that John is at the Continental and also believes that he believes that he is at the Holiday In.
- Then strong and weak interpretation differ: This explains Mary's reaction *and* can explain other reactions such as her going to the Continental despite believing that the original meeting was to take place at the Holiday In.

Conclusions

- (not surprising) Interpretation and belief as in ‘to believe’ are very distinct notions.
- There is a non-Gricean level of pragmatics that can be modeled directly on top of traditional representations of truth-conditional meanings computed from the lexicon.
- Once a hearer’s model of the speaker’s believes and its update is modeled, a number of fine-grained notions of belief, revision of iterated beliefs, and interpretation become available.

Open Issues

- How to implement soft update of the KD45 accessibility relation, i.e. when the agent takes into account things not previously considered, while maintaining the properties of the relation?
- Plausibility revision is categorical; is there a way to get more realistic graded revision?
- What's the relation between the above plausibility revision and AGM or KM belief revision/upgrade?