Making up one's mind

from values to value judgments

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Overview

The Idea

Represent values not just by their structure but by rules that relate preconditions to this structure:

$$P_1(x, y, \vec{z}), \ldots, P_n(x, y, \vec{z}) \mid\!\sim xR_iy \tag{1}$$

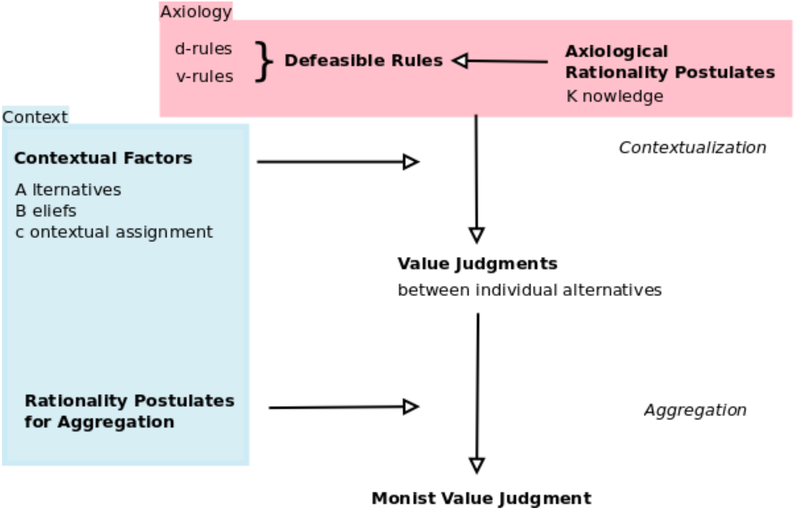
where

- x, y are variables for alternatives
- $\vec{z}$ are additional contextual ingredients
- $i \in N$; $N = \{1, 2, \ldots, n\}$ is the set of values or value attributes
- R_i is a relation between alternatives
- is a nonmonotonic inference relation

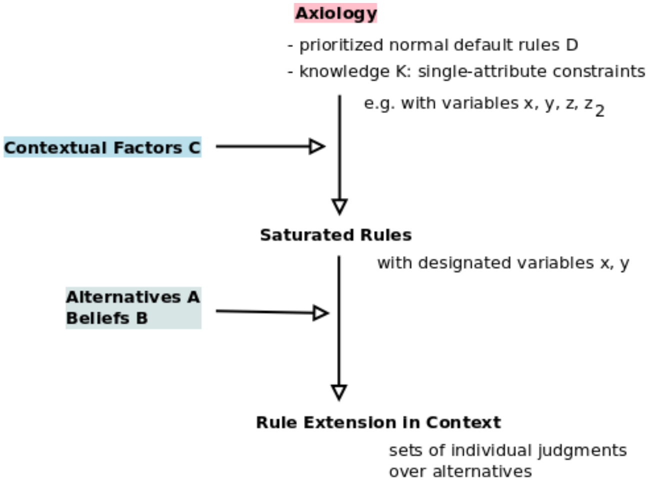
Why?

- The rationality requirements for single-attribute values differ from the rationality requirements for *applying* values.
- Values are applied for a given *purpose* such as decision making or comparing two or more particular alternatives, and comparing the values themselves. These call for different requirements.
- Values involve comparisons, e.g. ‘braver than’, ‘healthier than’, ‘better than’ are comparative forms.
- For a comparison to make sense some preconditions need to be fulfilled:
  - selectional criteria
  - relativization to contextual factors
  - features of the items under comparison
- These preconditions should be defeasible and prioritized.

The Process of Value Application



The Contextualization Step



Prioritized Default Rules

D-Rules

D-rules are of the form $P_1(\vec{x}_1) \;\&\; \cdots \;\&\; P_n(\vec{x}_n) \mid\!\sim C(\vec{x})$ and are open formulas of a (restricted) first-order default language $\mathcal{L}$.

V-Rules

V-rules are of the form $P_1(\vec{x}_1) \;\&\; \cdots \;\&\; P_n(\vec{x}_n) \mid\!\sim xR_iy$ where the antecedent conditions $P_1, \ldots, P_n$ are in $\mathcal{L}$, designated variables x and y must occur in at least one of them (non-vacuity), and R is one of $\|_i$, $\succ_i$, $\sim_i$ for attributes $1 \leq i \leq m$.

- The v-rule antecedents express defeasible conditions that must obtain in a context in order for two alternatives x, y to be judged xR_iy.
- All rules are ordered by a partial strict order $<$.

Why Defeasible Rules?

- More natural formulation: We do not want to codify all sort of exceptions in the rules.
- Contextual assumptions: We might want to retract some originally justified value judgments in light of more detailed knowledge of the context of application.

Deontic: (1) Do not kill unless in self-defense!

Axiological: (2) A situation in which you do not kill an aggressor is better than one in which you kill the aggressor unless the killing is necessary for self-defense.

Small print: This is just an example. I do not claim that there is a fixed connection between the deontic and axiological realms and do not subscribe to the Fitting Attitude Analysis of value.

A Less Value-laden Example

$\succ$

I prefer chocolate over vanilla ice cream...

A Less Value-laden Example

... unless the chocolate ice cream looks like this:

- We could also try to make all exceptions explicit.
- However, defeasible reasoning makes things easier.
- ~~However, defeasible reasoning makes things easier.~~ Not really.

The First Contextualization Step

In the current setting, the context dependence on additional factors ($\neq$ alternatives) is trivial from a logical point of view.

Contexts

A context $C = (A, B, c)$ consist of a set of alternatives A, some syntactic assignment function $c(.)$ mapping variables of type $\alpha > 0$ to terms of type α, and (if needed) a set of additional assumptions B.

Variables of type $\alpha > 0$ are replaced syntactically:

$$D' = \{\delta' \mid \delta' = \delta[{}^{x^\alpha}/{}_{c(\alpha)}] \text{ where } \delta \in D\} \tag{2}$$

Single Attribute Postulates

Conditions for single attributes $i \in N$:

$$\forall x . x \succeq_i x \tag{3}$$

$$\forall x, y, z . (x \succeq_i y \ \ \& \ \ y \succeq_i z) \rightarrow x \succeq_i z \tag{4}$$

$$\forall x, y . x \parallel_i y \rightarrow y \parallel_i x \tag{5}$$

$$\forall x, y . x \parallel_i y \rightarrow \neg(x \succeq_i y \vee y \succeq_i x) \tag{6}$$

Do we need default FOL?

- Full quantification: Use first-order default logic and add axiom schemes for single attributes (e.g. reflexivity, transitivity of $\succeq_i$).
  - Using Herbrand models: Kaminski (2011).
  - Embedding into S4F: Truszczyński (1991), Marek & Truszczyński (1993), Grimberg & Kaminski (2008).
- Finite domain:
  - Impose conditions by 'manually' computing reflexive and transitive closures and ensuring that $\|_i$ and $\succeq_i$ are distinct.
  - Default logic: Reiter (1980), prioritized version by Brewka & Eiter (1999).

The Second Contextualization Step

The rules are *normal* default rules with prerequisites.

General Rule	Normal Rule
$\dfrac{\alpha : \beta_1, \ldots, \beta_n}{\gamma}$	$\dfrac{\alpha : \gamma}{\gamma}$

$P_1(x, y), \ldots, P_n(x, y) \mid\!\sim C(x, y)$

- $\alpha : P_1(x, y), \ldots, P_n(x, y)$ set of prerequisites
- γ: $C(x, y)$ justification & conclusion

Closing of the defaults within a context:

Take into account all pairs of alternatives for which rules are *active*, i.e., those $\delta' = \delta[x/t_a, y/t_b]$ for which $pre(\delta') \subseteq (K \cup B)$ for terms $Tr(t_a) = a, Tr(t_b) = b$, contextually provided $a, b \in A$ and contextual assumptions B.

Example 1: Ice cream

C: chocolate ice cream
V: vanilla ice cream
S: 'soft ice'

$$\delta_1 : S(x), \neg S(y) \mathrel{|\!\sim} y \succ x$$
$$\delta_2 : C(x), V(y) \mathrel{|\!\sim} x \succ y$$
$$B = \{C(a), \neg C(b), V(b), \neg V(a), S(a), \neg S(b)\}$$
$$A = \{a, b\}$$

- Ordering $\delta_1 < \delta_2$: $B \cup \{b \succ a\} \subset E$
  Choose b over a.
- Ordering $\delta_2 < \delta_1$: $B \cup \{a \succ b\} \subset E$
  Chose a over b.

Example 2: The phone purchase

$$\delta_1 : Cheaper(x, y) \mid\!\sim x \succ_1 y$$
$$\delta_2 : Faster(x, y) \mid\!\sim x \succ_2 y$$
$$\delta_3 : Larger(x, y) \mid\!\sim x \succ_3 y$$
$$A = \{a, b, c\}$$
$$B = \{Cheaper(a, b), Cheaper(b, c), Faster(b, c), Faster(c, a), Larger(c, a), Larger(a, b)\}$$

This is a Condorcet case in separate attributes: abc, bca, cab (cf. Schumm 1987). The result depends on the aggregation step no matter how rules are prioritized.

Example 2: The phone purchase (cont'd)

$$\delta_1 : Cheaper(x, y) \mid\!\sim x \succ y$$
$$\delta_2 : Faster(x, y) \mid\!\sim x \succ y$$
$$\delta_3 : Larger(x, y) \mid\!\sim x \succ y$$
$$A = \{a, b, c\}$$
$$B = \{Cheaper(a, b), Cheaper(b, c), Faster(b, c), Faster(c, a), Larger(c, a), Larger(a, b)\}$$

In this case, the feature with the most preferred associated rule wins: $\{a \succ b, b \succ c\} \subset E$. Final aggregation is trivial.

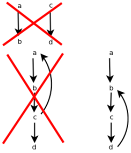
Connection to Social Choice

1. Decision Making: Function from partial preorders over alternatives to one or more winning alternatives. $\sim$ social choice function
2. 'Full Evaluation': Function from partial preorders over alternatives to an (incomplete preorder) relation over alternatives. $\sim$ social welfare function or weaker!
3. Pairwise Comparison: Only a pair of alternatives is provided contextually; function from individual value judgments to a value judgment. $\sim$ election with two candidates

However, partial preorders instead of complete linear orders are aggregated.

Criteria for the Outcome

1. Decision Making:
   1.1 Outcome must be weakly connected.
   1.2 Outcome may not have a top cycle.
2. 'Full Evaluation':
   2.1 Outcome must be weakly connected.
   2.2 Outcome must be cycle free.
3. Pairwise Comparison:
   3.1 The pair must be comparable in at least one attribute.



These criteria are fulfilled by Kemeny aggregation with 'minimal conflict merging' (to be explained) if every alternative is comparable in at least one attribute, i.e., $\forall a \in A \exists b \in A$ and $\exists i \in N$ s.t. $a \succeq_i b$ or $b \succeq_i a$.

Distance-based Aggregation: Kemeny

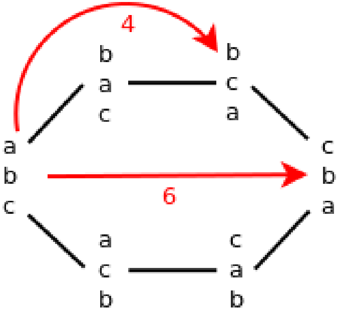
Kemeny's aggregation method (Kemeny 1959) determines that output ordering which minimizes the sum of the *inversion measures* between the input orderings. From a social choice perspective, it has various desirable properties:

- Neutrality: Result does not depend on the way alternatives are identified (switching positions in all input orderings leads to switched position in output).
- Unrestricted Domain: Any linear order is allowed as input.
- Pareto condition: If $a \succ_i b$ for all $i \in N$, then $a \succ b$.
- Consistency: If $a \succ_U b \ \& \ a \succ_V b$ for $U, V \subseteq N$, then $a \succ_{U \cup V} b$.
- Extended Condorcet: If for a partition V, U of A where $a \in U, b \in V$, and a majority $M \subseteq N$, $a \succ_M b$, then $U \succ V$.

Unfortunately, computation is NP-hard – but Spearman's footrule can be used as an approximation.

Kendall's Tau

Kendall's τ counts the number of inversions between two linear orders. Each node counts 2. The maximum distance is $\frac{1}{2}n(n+1)$.



There are several slightly distinct ways of dealing with ties of preorders. I average the results: $D(a\{bc\}d, b\{ad\}c) = \frac{1}{4}[D(abcd, badc) + D(abcd, bdac) + D(acbd, badc) + D(acbd, bdac)]$.

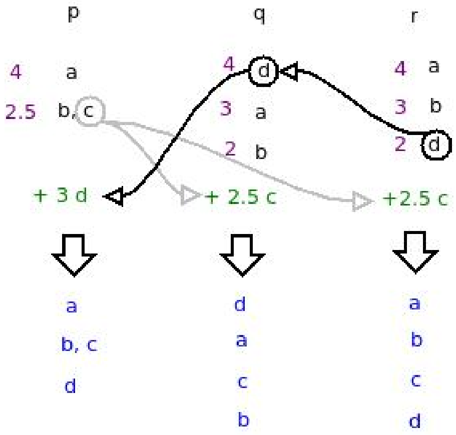
From Partial to Complete Preorders

1. Pini et al. (2011): Use all possible completions, i.e., those consistent with the base relation.
   - Pro: Accurately represents epistemic uncertainty.
   - Con: Does not reflect explicit noncomparability; high complexity.
2. Normalize ranks: Rank each relation and normalize to the number of items in its domain.
   - Pro: Not ad hoc and has fixed boundaries.
   - Con: Biases small preorders:

0.5	a	3	0.4	a	4
0.25	b, c	1.5	0.3	d	3
			0.2	c	2
			0.1	b	1

3. Normalize to largest domain: Divide the ranks by $K_{max} = \max(|X_i|)$ for all $i \in N$.
   - Pro: Avoids the bias of ordinary normalization.
   - Con: No unique sum, it depends on the size of the preorders.

My Suggestion: Top-Ranked Merging



Costs & Benefits

- Problems:
  - The approach is not very practical due to its complexity.
  - Currently the two components are completely separate.
  - Qualitative approach too limited.
- Benefits:
  - Explains many cases of value disagreement. Value disagreement is often not about the individual value judgments themselves but about the preconditions *or* their context.
  - This is disagreement about facts!

The Future...

There are a lot of things to do:

- Relax rationality postulates for single attributes.
- Philosophically justify the choice of nonmonotonic logic.
- Implement the default reasoning in a modal logic, including the context dependence.
- Investigate other modes of aggregation.
- Include cardinal constraints $u_i(x) - u_i(y) > k$.
- Use AAFs to model not only the default reasoning, but also argumentative attacks on preconditions in a multiagent setting.
- Add parity as a primitive relation? How would aggregation work in that case?

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Appendix A: Brief Comparison to Horty (2012)

Horty

- Defaults are for reasons that support other reasons or action-related conclusions.
- The outcome of reasoning is an intention to act, or a conflict between reasons remains.

Current Proposal

- Rule antecedents represent features of the alternatives under comparison, requirements for a value judgment.
- The outcome of the rule applications in a context is a set of individual value judgments that is in turn aggregated according to requirement called for by the purpose of comparing/judging alternatives. (two-step process)

Appendix B: Circularity Prohibition

No Circularity

Rule antecedents may not contain value relations (‘better than’ relations).

- This condition is purely conceptual; it cannot be formalized.

Appendix C: Which semantics?

- The union of all extensions yields inconsistent value judgments if there are two or more extensions.
- Incompatible value judgments like $a \succ_i b$ and $b \succ_i a$ should lead to $a \parallel_i b$ (or at best to $a \sim_i b$).
- We can already express this case, so an approach that yields *one* extension (if possible) seems desirable.
- In the current version of the paper I use a minor variant of prioritized default logic by Brewka & Eiter (1999) based on Reiter (1980).